it can be shown that any optimal strategy is asymmetric and non-step-wise in character for a corresponding choice of the numbers ρ and b.

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ONE SELFMODELLING SOLUTION OF A PROBLEM ON A PLANAR LAMINAR JET*

G.I. BURDE

The problem of the flow of a laminar jet which does not mix with the fluid surrounding it is treated in the boundary-layer approximation. It is assumed that both fluids are incompressible, that their surface of separation is smooth and that the jet does not break up. A selfmodelling solution (in Mieses variables) of the planar problem is obtained for the special case when the viscosities of the fluids are inversely proportional to their densities.

This problem has been treated previously in the case of a planar /1, 2/ and axially symmetric /3-7/ jet using different versions of the integral method /1, 3, 5, 7/ and also using an asymptotic method /2, 4, 6/ which yields the solution at large distances from the source.

1. The flow domain is shown schematically in Fig.1. Quantities referring to the emitted and external fluids are denoted by means of the indices 1 and 2. The equations of motion in the boundary layer approximation have the form

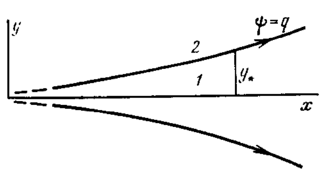

Fig.1

$$u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = \nu_i \frac{\partial^2 u_i}{\partial y^2}, \tag{1.1}$$

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0 \quad (i = 1, 2)$$

The conditions for the continuity of the velocities and the stresses on the boundary of separation in this approximation as well as the conditions on the axis of the jet and at infinity and the integral relationships expressing the laws of conservation of mass and momentum are represented in the form (only the upper half-plane is considered in view of the symmetry of the problem)

$$y = y_* (x),\ u_1 = u_2,\ \mu_1 \partial u_1/\partial y = \mu_2 \partial u_2/\partial y \tag{1.2}$$

$$y = 0,\ v_1 = 0,\ \partial u_1/\partial y = 0;\ y \to \infty,\ u_2 = 0$$

$$\int_0^{y_*(x)} \rho_1 u_1\, dy = \frac{Q}{2}, \quad \int_0^{y_*(x)} \rho_1 u_1^2\, dy + \int_{y_*(x)}^{\infty} \rho_2 u_2^2\, dy = \frac{J}{2} \tag{1.3}$$

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where Q and J are specified constant quantities (the flow rate and momentum of the jet).

We shall solve problem (1.1)-(1.3) in Mieses variables /8/: $\xi = x$, $\eta = \psi(x, y)$ where ψ is the stream function. Relationships (1.1)-(1.3) are then replaced by an equivalent system of equalities in the variables ξ and η

$$\frac{\partial u_i}{\partial \xi} = \nu_i \frac{\partial}{\partial \eta}\left(u_i \frac{\partial u_i}{\partial \eta}\right) \tag{1.4}$$

$$\eta = q,\ u_1 = u_2,\ \mu_1 \partial u_1/\partial\eta = \mu_2 \partial u_2/\partial\eta \tag{1.5}$$

$$\eta = 0,\ \partial u_1/\partial\eta = 0$$

$$\rho_1 q = \frac{Q}{2},\quad \rho_1 \int_0^q u_1\, d\eta + \rho_2 \int_q^{\eta_\infty(\xi)} u_2\, d\eta = \frac{J}{2} \tag{1.6}$$

Here q is the constant value of the stream function on the boundary of separation of the fluids, and $\eta_\infty(\xi)$ is the value of the stream function which corresponds to $y \to \infty$ and which is determined from the last condition of (1.2).

The relationships

$$y = \int_0^\eta \frac{dz}{u_1(x, z)},\quad \eta \leqslant q \tag{1.7}$$

$$y = \int_0^q \frac{dz}{u_1(x, z)} + \int_q^\eta \frac{dz}{u_2(x, z)},\quad \eta > q$$

are used to transform the solution to the initial variables. $\eta(x, y)$ can be found by inversion of these relationships.

2. We shall seek a selfmodelling solution of problem (1.4)-(1.6) in the form

$$u_i = \xi^{-1/3} f_i(w_i);\ w_i = \xi^{-1/3}(\eta + b_i) \tag{2.1}$$

where b_i are constants. Substituting (2.1) into Eq.(1.4), we obtain equations for f_i, the solutions of which are

$$f_i = C_i - (6\nu_i)^{-1} w_i^2$$

where C_i are constants. From these solutions and (2.1), we obtain

$$u_i = C_i \xi^{-1/3} - (6\nu_i \xi)^{-1}(\eta + b_i)^2 \tag{2.2}$$

Conditions (1.5) and (1.6) are used to determine the constants C_1, C_2, b_1 and b_2. It follows from the last condition of (1.5) that $b_1 = 0$. From the remaining conditions of (1.5) it follows that

$$C_1 = C_2,\ q^2/\nu_1 = (q + b_2)^2/\nu_2 \tag{2.3}$$

$$\mu_1 q/\nu_1 = \mu_2 (q + b_2)/\nu_2 \tag{2.4}$$

Conditions (2.3) and (2.4) are compatible if the relationship

$$\nu_2/\nu_1 = \mu_2^2/\mu_1^2,\ \text{or}\ \rho_1/\rho_2 = \mu_2/\mu_1 \tag{2.5}$$

is satisfied.

Assuming that condition (2.5) is satisfied, we determine the constant b_2 from (2.3) and find the solution in the form (we shall subsequently omit the index on the constant C)

$$u_1 = C\xi^{-1/3} - (6\nu_1\xi)^{-1}\eta^2 \tag{2.6}$$

$$u_2 = C\xi^{-1/3} - (6\nu_2\xi)^{-1}(\eta - q + q/\varkappa)^2,\ \varkappa = \sqrt{\nu_1/\nu_2}$$

The constant C is determined from the last condition of (1.6), to use which it is necessary to find the function $\eta_\infty(\xi)$. From the relationship $u_2(\xi, \eta_\infty) = 0$ we obtain

$$\eta_\infty = \sqrt{6C\nu_2}\,\xi^{1/3} + q(1 - \varkappa^{-1}) \tag{2.7}$$

Substituting (2.6) into (1.6) and using (2.7) and (2.5), we arrive, after some reduction, at the relationship

$${}^2/_3 \rho_2 \sqrt{6\nu_2}\, C^{3/2} = {}^1/_2 J \tag{2.8}$$

Eqs.(2.6) and (2.8) and the first equation of (1.6) solve the problem which has been posed in the variables ξ, η. Let us obtain the solution in the initial variables using formulae (1.7). First we write out the expression for the half-width of the jet

$$y_*(x) = \sqrt{\frac{3\nu_1}{2C}}\, x^{2/3} \ln \frac{\sqrt{6C\nu_1}\, x^{1/3}}{\sqrt{6C\nu_1}\, x^{1/3} - q} \tag{2.9}$$

When $y < y_*$ (the fluid in the jet) we find $y(x, \eta)$ from the first formula of (1.7) and, by inverting the resulting expression, we arrive at the formula

$$\eta_1 = \sqrt{6C\nu_1}\, x^{1/3} \frac{e^{\zeta} - 1}{e^{\zeta} + 1}; \quad \zeta = \sqrt{\frac{2C}{3\nu_1}} \frac{x}{x^{2/3}} \tag{2.10}$$

Substituting (2.10) into the first formula of (2.6) or differentiating (2.10) with respect to y we find

$$u_1 = 4Cx^{-1/3} e^{\zeta} (e^{\zeta} + 1)^{-2} \tag{2.11}$$

In a similar way but using the second formula of (1.7), we obtain, when $y > y_*$,

$$\eta_2 = \sqrt{6C\nu_2}\, x^{1/3} \frac{\varphi(x) e^{\varkappa\zeta} - 1}{\varphi(x) e^{\varkappa\zeta} + 1} + q(1 - \varkappa^{-1}) \tag{2.12}$$

$$\varphi(x) = [(\sqrt{6C\nu_1}\, x^{1/3} + q)/(\sqrt{6C\nu_1}\, x^{1/3} - q)]^{1-\varkappa}$$

Whence, we find

$$u_2 = 4Cx^{-1/3} \varphi(x) e^{\varkappa\zeta} [\varphi(x) e^{\varkappa\zeta} + 1]^{-2} \tag{2.13}$$

Expressions for the second component of the velocity v can be obtained from (2.10) and (2.12) using the relationship $v = -\partial\eta/\partial x$.

Hence, when condition (2.5) is satisfied, formulae (2.8)-(2.13) and the first formula of (1.6) yield a solution of the problem, which one can confirm by directly substituting this solution into Eqs.(1.1)-(1.3). By considering the asymptotic form of this solution when $x \gg 1$ it is possible to obtain expressions which are identical to those given in /2/ subject to condition (2.5).

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